# Multi-Objective Rectangular Packing Problem and Its Applications

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**Abstract.** In this paper, Neighborhood Cultivation GA (NCGA) is applied to the rectangular packing problem. NCGA is one of the multi-objective Genetic Algorithms that includes not only the mechanisms of effective algorithms such as NSGA-II and SPEA2, but also the mechanism of the neighborhood crossover. This model can derive good non-dominated solutions in typical multi-objective optimization test problems. The rectangular packing problem (RP) is a well-known discrete combinatorial optimization problem in many applications such as LSI layout problems, setting of plant facility problems, and so on. The RP is a difficult and time-consuming problem since the number of possible placements of rectangles increase exponentially as the number of rectangles increases. In this paper, the sequent-pair is used for representing the solution of the rectangular packing and PPEX is used as the crossover. The results were compared to the other methods: SPEA2, NSGA-II and non-NCGA (NCGA without neighborhood crossover). Through numerical examples, the effectiveness of NCGA for the RP is demonstrated and it is found that the neighborhood crossover is very effective both when the number of modules is small and large.

## 1 Introduction

Recently, the study of the evolutionary computation of multi-objective optimization has been actively researched and has made great progress [1, 9]. The genetic algorithm (GA) is standout among the many approaches that have been proposed [1]. Since GA is one of multi point search methods, it can approximate a set of Pareto-optimum solutions in a trial. This phenomenon is one of the reasons why GA is studied in the field of multi-objective optimization problems.

In recent years, several new algorithms that find good Pareto-optimum solutions with small calculation costs have been developed [1]. They are NSGA-II [1], SPEA2 [9], and so on. These new algorithms have the same search mechanisms:

the preservation scheme of excellent solutions found in the search and the sharing scheme without parameters.

On the other hand, we proposed the new model of multi-objective GA that called Neighborhood Cultivation GA (NCGA) [8]. NCGA includes not only the mechanisms of NSGA-II and SPEA2 that derive the good solutions but also the mechanism of neighborhood crossover.

This model derives good Pareto solutions in typical multi-objective optimization test problems. From the results of the test functions, it is theorized that NCGA can derive good solutions in complicated problems like large-scale or real-world problems.

In this paper, NCGA is applied to the rectangular packing problem (RP). Because RP is a NP-hard problem, good heuristics are generally solicited. The RP can be found in a setting problem of LSI floor plan problem [4–6], plant facilities [7], and so on. The sequent-pair is used for representing a solution of the rectangular packing and PPEX is used as a crossover.

In numerical experiments, the results of NCGA are compared with those of NSGA-II, SPEA2 and non-NCGA. Non-NCGA is the same algorithms as NCGA but without neighborhood crossover.

## 2 Multi-Objective Optimization Problems by Genetic Algorithms and Neighborhood Cultivation GA

### 2.1 Multi-Objective Optimization Problems and Genetic Algorithm

Several objectives are used in multi-objective optimization problems. These objectives usually cannot be minimized or maximized at the same time due to a trade-off relationship between them [1]. Therefore, one of the goals of the multi-objective optimization problem is to find a set of Pareto-optimum solutions.

The Genetic Algorithm is an algorithm that simulates the heredity and evolution of living things [1]. Because it is a multi point search method, an optimum solution can be determined even when the landscape of the objective function is multi modal. It can also find a Pareto-optimum set with one trial in multi-objective optimization. As a result, GA is a very effective tool for multi-objective optimization problems. Many researchers are researching multi-objective GA and are developing many algorithms of multi-objective GA [1–3, 9].

The algorithms of multi-objective GA are roughly divided into two categories: algorithms that treat Pareto-optimum solutions implicitly and algorithms that treat Pareto-optimum solutions explicitly [1]. Many of the latest methods treat Pareto-optimum solutions explicitly.

The typical algorithms that treat Pareto-optimum solutions explicitly are NSGA-II [1] and SPEA2 [9]. These algorithms have the following similar schemes:

1) Reservation mechanism of the excellent solutions
2) Reflection to search solutions mechanism of the reserved excellent solutions
3) Cut down (sharing) method of the reserved excellent solutions
4) Unification mechanism of values of each objective

These mechanisms derive the good Pareto-optimum solutions. Consequently, the developed algorithms should have these mechanisms.

### 2.2 Neighborhood Cultivation Genetic Algorithm

In this paper, we extend GA and develop a new algorithm called Neighborhood Cultivation Genetic Algorithm (NCGA). NCGA has the neighborhood crossover mechanism in addition to the mechanisms of GAs that are explained in the former chapter. In GAs, exploration and exploitation are very important. By exploration, an optimum solution can be found around the elite solution. By exploitation, an optimum solution can be found in a global area. In NCGA, the exploitation factor of the crossover is reinforced. In the crossover operation of NCGA, a pair of the individuals for crossover is not chosen randomly, but individuals who are close to each other are chosen. Because of this operation, child individuals that are generated after the crossover may be close to the parent individuals. Therefore, the precise exploitation is expected.

The following steps demonstrate the overall flow of NCGA where

$P_t$ : search population at generation
$A_t$ : archive at generation .

Step 1: Initialization: Generate an initial population $P_0$. Population size is $N$. Set $t = 0$. Calculate fitness values of the initial individuals in $P_0$. Copy $P_0$ into $A_0$. Archive size is also $N$.
Step 2: Start new generation: set $t = t + 1$.
Step 3: Generate new search population: $P_t = A_{t-1}$.
Step 4: Sorting: Individuals of $P_t$ are sorted according to the values of the focused objective. The focused objective is changed at every generation. For example, when there are three objectives, the first objective is focused in the first generation and the third objective is focused in the third generation. The first objective is focused again in the fourth generation.
Step 5: Grouping: $P_t$ is divided into groups consisting of two individuals. These two individuals are chosen from the top to the bottom of the sorted individuals.
Step 6: Crossover and Mutation: In a group, crossover and mutation operations are performed. From two parent individuals, two child individuals are generated. Here, parent individuals are eliminated.
Step 7: Evaluation: All of the objectives of individuals are derived.
Step 8: Assembling: All the individuals are assembled into one group and this becomes new $P_t$.
Step 9: Renewing archives: Assemble $P_t$ and $A_{t-1}$ together. The $N$ individuals are chosen from $2N$ individuals. To reduce the number of individuals, the same operation of SPEA2 (Environment Selection) is performed. In NCGA, this environment selection is applied as a selection operation.
Step 10: Termination: Check the terminal condition. If it is satisfied, the simulation is terminated. If not, the simulation returns to Step 2.

In NCGA, most of the genetic operations are performed in a group consisting of two individuals.

The following features of NCGA are the differences from SPEA2 and NSGA-II.

1) NCGA has the neighborhood crossover mechanism.
2) NCGA has only the environment selection. It does not have the mating selection.

## 3 Formulation of Layout Problems and Configuration of Genetic Algorithm

The rectangular packing problem (RP) is a well-known discrete combinatorial optimization problem in many applications such as LSI layout problems [4–6], plant facilities [7], and so on.

The rectangular packing problem (RP) is known to be a difficult and time-consuming problem since the number of possible placements of rectangles increase exponentially as the number of rectangles increases.

A module(block) $m_i \in M, (0 \leq i < n)$ is a rectangle with a given height and width in real numbers. A packing of a set of modules is a non-overlapping placement of given modules. The problem of RP is to find a packing $M$ with the minimum area. This problem is NP-hard; therefore, good heuristics are generally solicited.

In this paper, we treat the RP as two objective optimization problems. This multi-objective RP aims to minimize not the packing area but the width and height of the packing area. In this formulation, a decision maker can select the aspect ratio of packing area.

### 3.1 Genetic Approach for RP

**Representation** Many approaches have been proposed to solve RP in a practical computation time [4, 5]. One important key in the struggle to solve the problem is the representation of an instance of RP. Recently, sequence-pair [4] and BSG [5] have been proposed as a solution of this problem. Sequence-pair and BSG are particularly suitable for stochastic algorithms such as GA and simulated annealing(SA). These coding schemes can represent not only slicing structure but also non-slicing structure. Currently, these coding schemes are different reverse polish notation (RPN).

In this paper, we used sequence-pair as the representation of a solution, since sequence-pair can perform more effective searches than BSG. The number of all combination of sequence-pair is smaller than that of BSG.

**Sequence-Pair** The sequence-pair is used for representing the solution of the rectangular packing. Each module has the sequence-pair $(\Gamma_{-}, \Gamma_{+})$. By comparing the sequence-pair of the two modules, the relative position of these modules are

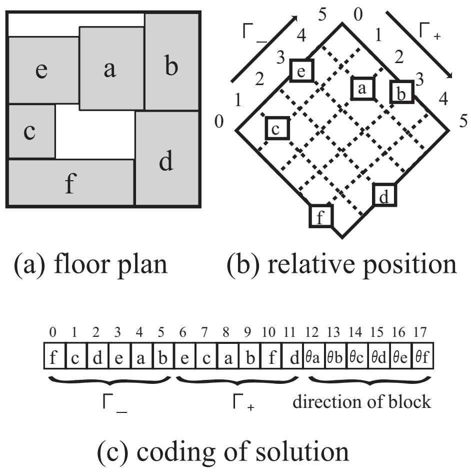


**Fig. 1.** Coding example of sequence-pair.

defined. Let module $A$ and $B$ have the sequence pairs $(x_{a-}, y_{a+})$ and $(x_{b-}, y_{b+})$ respectively. In this case, there is a relationship between the positions of the modules and the sequence pairs as follows,

$$\begin{array}{c}\text{when } x_{a-} < x_{b-} \text{ and } y_{a+} < y_{b+}, A \text{ is in the left side of } B \\ \text{when } x_{a-} > x_{b-} \text{ and } y_{a+} > y_{b+}, A \text{ is in the right side of } B \\ \text{when } x_{a-} < x_{b-} \text{ and } y_{a+} > y_{b+}, A \text{ is in the upper side of } B \\ \text{when } x_{a-} > x_{b-} \text{ and } y_{a-} < y_{b+}, A \text{ is in the bottom side of } B.\end{array}$$

In addition to the sequence-pair, each module has the orientation information $\Theta$. This information instructs the direction of the module arrangement.

**Coding System** A gene of the GA consists of three parts; those are $\Gamma_-$, $\Gamma_+$, and $\Theta$. Fig. 1 displays the coding for 6 modules.

From the coding information (Fig. 1(c)), the relative position (b) is derived. This position shows the floor plan (a). In this paper, each module is settled lengthwise or breadwith. Therefore, $\Theta$ takes 0 or 1.

**Crossover Operator** In this paper, we use the Placement-based Partially Exchanging Crossover (PPEX) [6] that was proposed by Nakaya and et al. The PPEX makes a window-territory that locates in the neighborhood of modules chosen randomly. This window-territory means a continuous part of the oblique-grid that is defined by the sequence-pair. The PPEX performs as a crossover that exchanges modules within this window-territory. Therefore the PPEX can exchange modules within the neighborhood position. The procedure of the PPEX is illustrated as follows.

Step 1: Two modules are chosen randomly as parent modules.

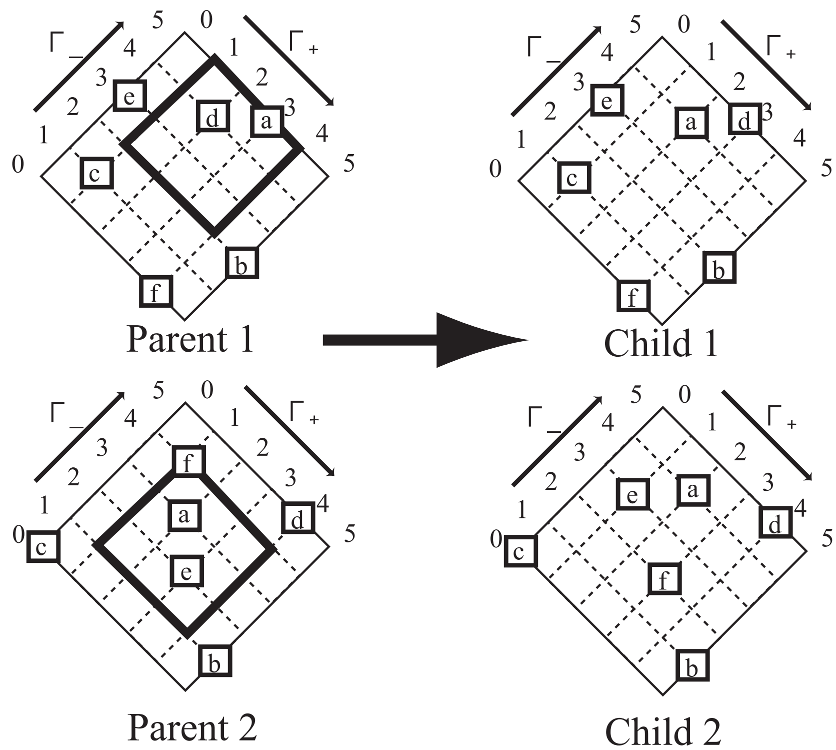


**Fig. 2.** Placement-based Partially Exchanging Crossover (PPEX).

Step 2: The window-territory is created in the neighborhood of the chosen modules. Let $M_c$ be the set of modules within window-territory and $M_{nc}$ be the set of the rest modules.
Step 3: Each module of $M_c$ is exchanged according to the sequence of its partner parent and is copied to the child.
Step 4: $M_{nc}$ are directly copied to child.

Fig.2 displays PPEX when the window-territory size is 4.

In Parent 2, modules of $a$ and $e$ are chosen for $M_c$. Modules of $M_c$ are exchanged. In this exchange, the relative position of the other parent is referenced. Then these modules are copied to the child. With the location information of Parent 1, $a$ $e$ and $f$ are moved then copied to child 2.

**Mutation Operator** In this paper, we use bit flip of orientation for module($\theta$). That is, if $\theta$ is 1, it let $\theta$ be 0, the opposite, if $\theta$ is 0, it let $\theta$ be 1.

### 3.2 Formulation of Layout Problems

In this paper, there are two objectives as follows,

**Table 1.** GA Parameters

| | |
|---|---|
| population size | 200 |
| crossover rate | 1.0 |
| mutation rate | 1/bit length |
| terminal generation | 400 |

$$\min f_1(x) = \text{width of packing area of modules}$$
$$\min f_2(x) = \text{length of packing area of modules}$$

These two objects have trade-off relations with each other. A decision maker can select an aspect ratio of packing area.

## 4 Numerical Examples

In this paper, NCGA is applied to some numerical experiments. We used four instances of this problem: ami33, ami49, pcb146 and pcb500. The instances ami33 and ami49 whose data are in the MCNC benchmark consist of 33 and 49 modules (rectangles). The instances pcb146 and pcb500 were given by Kajitani [4]. These instances have 146 and 500 rectangles, respectively.

The results are compared with those of SPEA2 [9], NSGA-II [1] and non-NCGA. Non-NCGA is the same algorithm of NCGA without the neighborhood crossover.

### 4.1 Parameters of GAs

Table. 1 displays the used GA parameters. We used the above GA operator, PPEX and the bit flip of module orientation. The length of the chromosome is three times as long as the number of modules.

### 4.2 Evaluation methods

To compare the results derived by each algorithm, the following evaluation methods are used.

**Sampling of the Pareto frontier lines of intersection($I_{LI}$)** This comparison method is presented by Knowles and Corne [3]. The concept of this method is shown in Fig. 3. This figure illustrates two solution sets of X and Y derived by the different methods.

At first, the attainment surfaces defined by the approximation sets are calculated. Secondly, the uniform sampling lines that cover the Pareto tradeoff area are decided. For each line, the intersections of the line and the attainment surfaces of the derived sets are obtained. These intersections are compared. Finally,

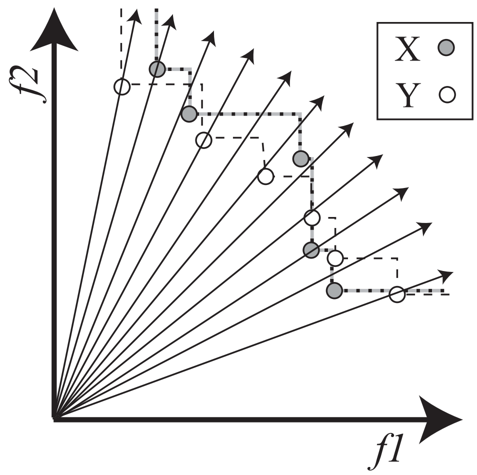


**Fig. 3.** Sampling of the Pareto frontier lines of intersection.

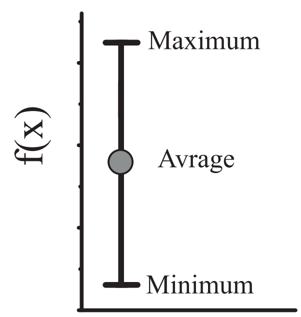


**Fig. 4.** An example of $I_{MMA}$.

the Indication of Lines of Intersection ($I_{LI}$) is derived. When the two approximation sets $X$ and $Y$ are considered, $I_{LI}(X, Y)$ indicates the average number of the points $X$ are ranked higher than $Y$. Therefore the most significant outcome would be $I_{LI}(X, Y) = 1.0$ and $I_{LI}(Y, X) = 0.0$.

To focus only on the Pareto tradeoff area as defined by the approximation sets and to derive the intuitive evaluation value, the following terms are considered:

- The objective values of approximation sets are normalized.
- The sampling lines are located in the area where the approximation sets exist.
- Many sampling lines are prepared. In the following experiment, 1000 lines are used.

**Maximum, Minimum and Average values of each object of derived solutions ($I_{MMA}$)** To evaluate the derived solutions, not only the accuracy but also the expanse of the solutions is important. To discuss the expanse of the

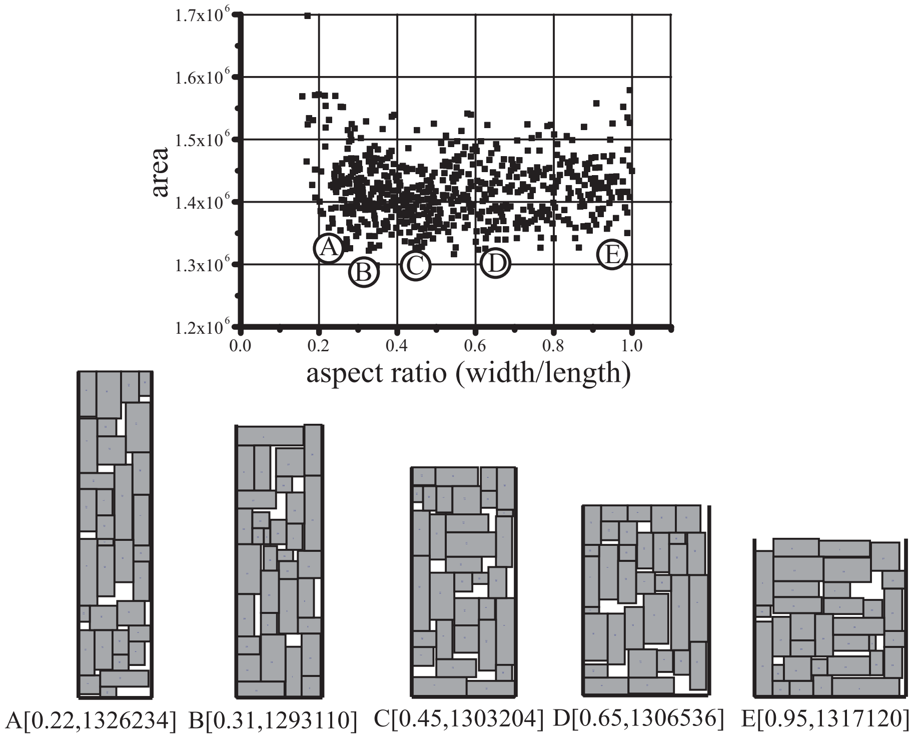


**Fig. 5.** The placement of the modules(ami33).

solutions, the maximum, minimum and average values of each object are considered. Figure 4 is an example of this measurement. In this figure, the maximum and minimum values of objective function are illustrated. At the same time, the medium value is shown as a circle.

### 4.3 Results

In this study, we tried four types of a problem: ami33, ami49, pcb146 and pcb500 modules. In this section, we discuss only the instances ami33 and pcb500.

Proposed NCGA, SPEA2, NSGA-II and non-NCGA (NCGA without neighborhood crossover) are applied to these problems. 30 trials have been performed and all the results are the average of the 30 trials.

**Layout of the solution** It should be verified whether solutions that are derived by the algorithm are opposite placement of modules. In this section, we focus on the ami33 which consist of 33 modules. The placement of ami33, which is presented by solutions of NCGA, is shown in Fig. 5.

In Fig. 5, some of the typical solutions are illustrated. Since this is the combination of $N! \times N! \times 2^N$ problem with $N$ module, the real optimum solutions

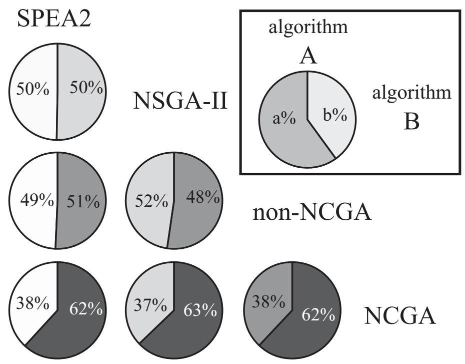


**Fig. 6.** Results of $I_{LI}$(ami33).

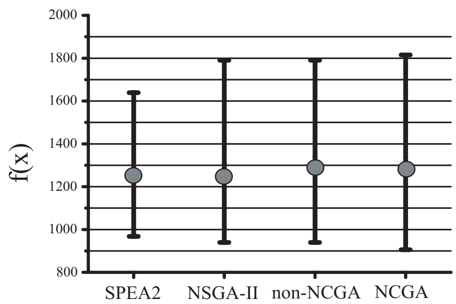


**Fig. 7.** $I_{MMA}$ of ami33

are not derived. In this paper, 80,000 function calls (200 population and 400 generations) are performed. These results may be reasonable, since there are very few blank spaces. We also use a sequence-pair and PPEX to derive good solutions since these techniques are very suitable for GA and RP.

**ami33** In the results of ami33, $I_{LI}$ are shown in Fig. 6, and $I_{MMA}$ are shown in Fig. 7 . Fig. 8 shows the nondominated solutions of each algorithms. In this figure, all nondominated solutions derived from the 30 trials are plotted.

$I_{LI}$ of Fig. 6 indicates that solutions of NCGA are closer to the real Pareto solutions than those of the other methods. This fact is also given from the plots of the nondominated solutions(Fig.8). It is also clear from $I_{MMA}$ of Fig. 7 that NCGA and non-NCGA can find the wide spread nondominated solutions compared to the other methods.

Non-NCGA can get wide-spread nondominated solutions. However, compared to the real Pareto solutions, non-NCGA is not ideal. This result shows that the neighborhood crossover derives good solutions in RP.

**pcb500** The results of pcb500 are shown in Fig. 9 and Fig. 10. Fig. 11 illustrates the nondominated solutions of the different algorithms.

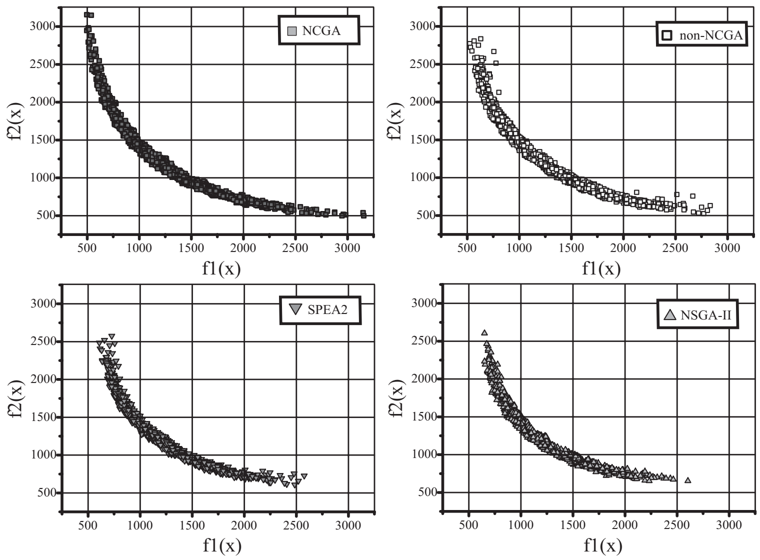


**Fig. 8.** Nondominated solutions(ami33).

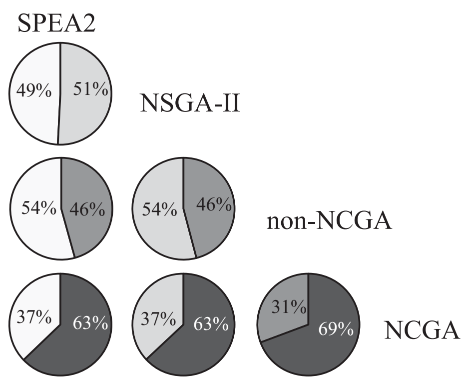


**Fig. 9.** Results of $I_{LI}$(pcb500)

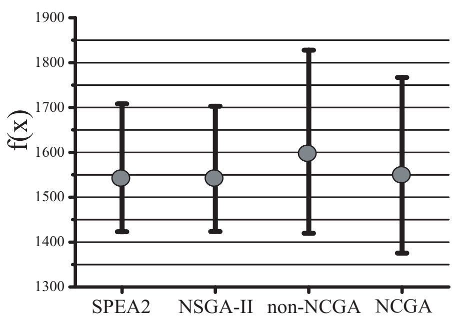


**Fig. 10.** $I_{MMA}$ of pcb500

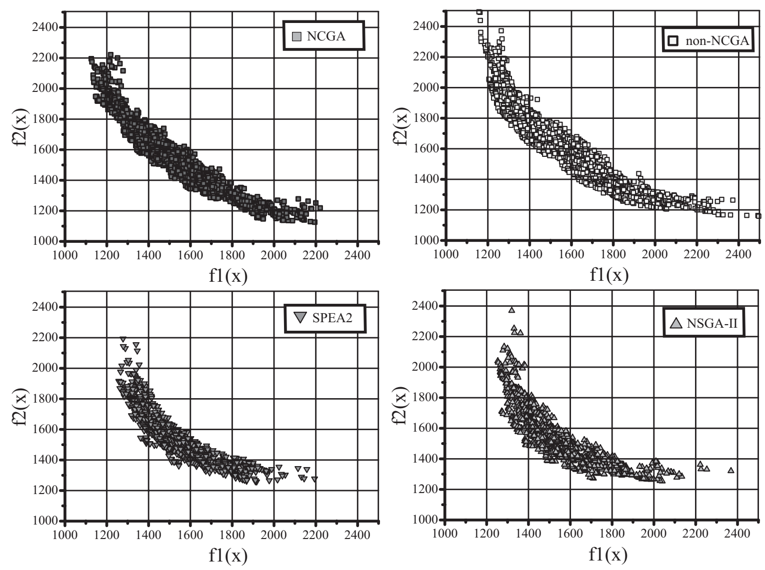


**Fig. 11.** Nondominated solutions(pcb500).

The tendency of the results from this problem is similar to those of the previous problem. From Fig.9 and Fig.11, it is clear that NCGA obtained the better value of $I_{LI}$; namely, the solution of NCGA is much better than those of the other. Like the previous problem, the solutions of non-NCGA are so far from the real Pareto front. Therefore the neighborhood crossover is very effective to derive the good solutions in RP, irrespective of the number of modules.

On the other hand, in this problem, the solutions of SPEA2 and NSGA-II are gathered around the center of the Pareto front. These results emphasize that SPEA2 and NSGA-II tend to concentrate in one part of the Pareto front when the number of modules is very large. On the other hand, Fig10 and Fig11 indicate that NCGA and non-NCGA could keep the high diversity of the solution during the search even if the number of modules is very large.

## 5 Conclusion

In this paper, Neighborhood Cultivation GA (NCGA) is applied to the rectangular packing problem (RP). NCGA has not only the important mechanism of the other methods but also the mechanism of the neighborhood crossover selection. In this paper, the sequent-pair is used for representing a solution of the rectangular packing. PPEX is also used as a crossover.

To discuss the effectiveness of NCGA to RP, NCGA was applied to RP and its results were compared to the other methods: SPEA2, NSGA-II and non-NCGA (NCGA without neighborhood crossover). Through numerical examples, the following topics are clarified.

1) The RP that is used in this paper is a large scale problem. For this problem, a reasonable solution is derived with a small calculation cost. It is assumed that a sequence-pair and PPEX work well in this problem.
2) In almost all the test functions, the results of NCGA are superior to that of the others. From this result, it can be noted that NCGA is a good method for the RP.
3) Comparing NCGA and NCGA without the neighborhood crossover, the former is obviously superior to the latter in all the problems. The results emphasize that the neighborhood crossover acts to derive the good solutions in the RP.
4) When the number of modules is very large, the solutions of SPEA2 and NSGA-II tend to concentrate in the center of the Pareto front. However, NCGA and non-NCGA could keep the diversity of the solutions.

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