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Stress-based Crossover Operator for Structural Topology Optimization*

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Abstract
In this paper, we propose a stress-based crossover (SX) operator to solve the checkerboard-like material distributation and disconnected topology that is common for simple genetic algorithm (SGA) to structural topology optimization problems (STOPs). A penalty function is defined to evaluate the fitness of each individual. A number of constrained problems are adopted to experiment the effectiveness of SX for STOPs. Comparison of 2-point crossover (2X) with SX indicates that SX can markedly suppress the checkerboard-like material distribution phenomena. Comparison of evolutionary structural optimization (ESO) and SX demonstrates the global search ability and flexibility of SX. Experiments of a Michell-type problem verifies the effectiveness of SX for STOPs. For a multi-loaded problem, SX searches out alternate solutions on the same parameters that shows the global search ability of GA.

Key words : Genetic Algorithm, Stress-based Crossover, ESO, Structure Topology Optimization, Structure Optimization

1. Introduction

Continuum structural topology optimization involves searching for an optimal material layout. Cheng and Olhoff[1] first reported that optimization techniques using spatial distributions of design variables can change and even optimize the topology of material distributions in a structure. Structural topology optimization via distributed parameter optimization techniques was first proposed by Kohn and Strang[2]. Bendsϕe and Kikuchi reported the homogenization method in 1988[3], in which each element in a grid contains composite material of continuously variable density [0,1] and orientation. However, evaluation of optimal microstructures and their orientations is usually cumbersome and numerically complicated[4]. As an important alternative approach, the SIMP (Solid Isotropic Microstructure with Penalization) method[5] was introduced and has gained general acceptance in recent years because of its computational efficiency and conceptual simplicity. However, it does not directly deal the original 0-1 problems[6] and thus tends to converge to a local optimal topology with blurry boundaries or undesirable checkerboard patterns[6],[7] or to converge to an infeasible solution to the original 0-1 problems[8]. Xie and Steven proposed the ESO[9] method that follows the concept of gradual removal of inefficient material from a structure. However, the ESO method is not based on the principle of optimization algorithm, and may also easily lead to a non-optimal design[7],[10]. As an extension of the ESO method, the bidirectional evolutionary structural optimization (BESO) method allows efficient materials to be added in addition to removal of inefficient materials to remedy the elements deleted in previous processes[11]. However, the rejection ratio and inclusion ratio used in BESO indicated that these ratios were dependent on a number of other properties[12]. Furthermore, it is questionable to extend these approaches to other design cases, such as multi-physics problems and mul-

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tiple constrained problems. Recently, evolutionary computation methods, such as genetic algorithm (GA), evolution strategies, and evolution programming, have been used to solve various engineering problems. Especially, GA has been verifed a global search algorithm and applied to various complicated engineering problems. However, for SGA to structural topology problems, the binary genotype and 1-point/2-point crossover operators often cause checkerboard-like patterns(13),(14) and disconnected topologies(15), which make the solution impractical. Diazand and Sigmend(16), Jog and Haber(17) have shown that the checkerboard pattern are due to bag numerical modeling of the stiffness of checkerboards. Both works demenstrate checkerboard patterns are prone to appear in both the hompgenization and the SIMP approach. O.Sigmund and J.Petersson(18) gave a survey on procedures dealing with checkerboards, mesh-dependecies and local minima in topology optimization. To suppress the checkerboard patterns, Li and Steven(19) proposed an effective smoothing algorithm in terms of the surrounding elementsreference factors. Sigmund(20) suggested a checkerboard prevention filter. Some researchers focus on chromosome representations. Therefore, bit-string/bit-array, graph representation(21) and morphological geometric representation(22) were introduced to guarantee the mesh connectivity. Furthermore, image processing based filtering is used to suppress the occurrence of checkerboard patterns(23). Kim and Week introduced a variable chromosome length genetic algorithm in topology optimization(24). Inspired by the ESO algorithm, we propose a stress-based crossover operator, which seeks to combine the advantages of ESO and GA.

2. Structural Topology Optimization by ESO and GA

2.1. ESO

ESO is one formal approach for STOPs, which is based on the concept of removing less stressed elements gradually. In ESO, there are two important parameters: rejection rate (*rrRate*) and evaluation rate (*erRate*). The element with stress meeting formula (1) will be deleted in every generation. Here, $Stress^k$ is the stress of element-k. If there is no element deleted at the current *rrRate*, it is adjusted by formula (2). According to experiments, these two parameters are usually assigned small values initially.

$$Stress^k / Stress_{max} < rrRate \tag{1}$$

$$rrRate = rrRate + erRate \tag{2}$$

2.2. GA to Structural Topology Optimization

GA is based on the principles of evolution and natural selection. According to the schema theory and elite save mechanism, GA is verified as an algorithm with global search capability that definitely converge to an optimum. GA to STOPs include the following main procedures: initial population generation, FEA on population, selection, crossover, mutation, individual evaluation, and termination decision.

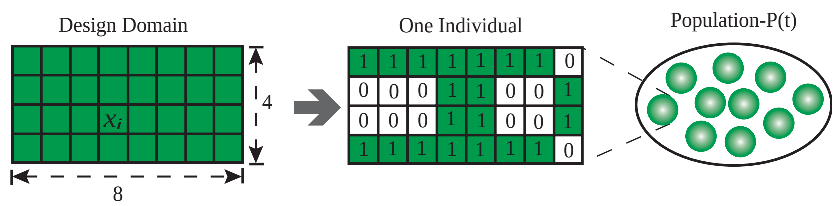


Fig. 1 Design variable and chromosome representation

2.2.1. Design Variable and Chromosome Representation For GA to STOPs, straight-forward chromosome representation is bit-string or bit-array. As shown in Fig.1, the structure is taken as one individual. The design domain is divided into fixed regular meshes. Each mesh (which is also called an element in FEA) represents one gene of the chromosome with 1 for solid material and 0 for void material. Each mesh is one design variable - x_i. For example, in Fig.1 the design domain is divided into 4×8=32 meshes, therefore there are 32 variables. In this study, the bit-string chromosome representation is adopted.

3. Stress-based Crossover Operator

By now, GA with 1-point/2-point crossover and uniform mutation operator often derives a solution with checkerboard pattern or a solution with disconnected phenomena that make the result impractical. One reason for these problems is that neighboring meshes continuity is not considered sufficiently because properties of structure, such as stress and stiffness, often vary gradually. ESO, which can obtain a stress-balanced topology from starting rejection from a full material structure according to the element stress, has been extensively used to solve various problems. In this paper, we propose a stress-based crossover operator to solve the "checkerboard" problems. The procedures of this operator is also illustrated in Fig. 2. First, the nomenclature used in this operator is explained.

- P(t): Population of generation t.
- $p_i(t)$: One individual.
- $p_i(t).weight$: Number of "1" in $p_i(t)$.
- $p_i(t).code[k]$: One gene of $p_i(t)$, $k = 1 \ldots N$.
- $p_i(t).stress[k]$: Element stress of $p_i(t)$.
- $p'_i(t).ability[k]$: Ability value of $p'_i(t)$.

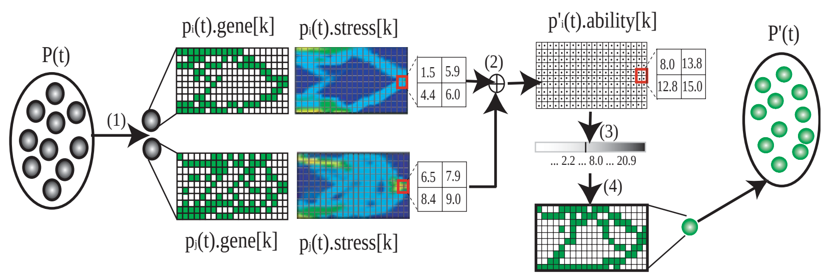


Fig. 2 SX Operator Procedures

(1) *Randomly select two individuals,* $p_i(t)$, $p_j(t)$ *from population* $P(t)$.

(2) *Add up the stress at each gene of* $p_i(t)$ *and* $p_j(t)$ *by formula(3). Naming this value as the ability of each gene of child individual.*

$$p'_i(t).ability[k] = p_i(t).stress[k] + p_j(t).stress[k], k = 1 \ldots N \quad (3)$$

(3) *Sort the ability values of child individual* $p'_i(t).ability$ *from big to small.*

(4) *According to the ability value of each gene, the bigger ability valued genes will be set "1", others will be set "0". Namingly, divide the genes into two groups, U1 and U0. U1 is group of the genes that is set "1". U0 is group of the genes that is set "0". The new generated individual is named* $p'_i(i)$. *In this study,* $p'_i(t).weight$ *is defined by formula (4). Generate a child individual-*$p'(t)$ *by formula (5).*

$$p'_i(t).weight = [p_i(t).weight + p_j(t).weight]/2 \quad (4)$$

$$p'_i(t).code[k] = \begin{cases} 1, & if\ p'_i(t).ability[k] \in U1 \\ 0, & if\ p'_i(t).ability[k] \in U0 \end{cases} \quad (5)$$

In the above four steps, in step (4) how to define the "1" number of the child individual - $p'_i(t)$ is the key point. It is known that for the initial individuals disconnected phenomena and checkerboard-like material distribution are common because of the bitstring chromosome representation and randomly generation method. Furthermore, for some strick constrained problems, certain weighted structure is required. In another word, the "1" number in the chromosome must be more that certain number. Therefore, in this SX operator, we want the the "1" number of chromosome changed gradually. Hence, the "1" number of the child individual is defined as formula (4).

One child individual is generated after the above four steps. Applying these four steps on the population - $P(t)$ to generate child population - $P(t+1)$. The child population - $P(t+1)$ will replace the population - $P(t)$ and reserve to next generation.

After SX operation, the mutation operator is applied on each gene of each individual with a small rate. The mutation operator focuses on local search. Randomly decreasing "1" number in the chromosome drives a lighter topology. Increasing "1" number in the chromosome may remedy the infeasible individual.

4. Objective Function and Fitness Function

In this paper, the objective function is to minimize the weight subject to constrained stress, $Stress_{lim}$, and constrained displacement, $Disp_{lim}$, that is formulated as (6). Where, $Stress_{max}$ is the maximal stress of the structure and $Disp_{max}$ is the maximal displacement of the loading point.

$$\begin{aligned} min.f(X) &= \sum_{i=1}^{N} x_i, x_i \in \{0,1\} \\ subject\_to: Stress_{max} &< Stress_{lim} \\ Disp_{max} &< Disp_{lim} \end{aligned} \tag{6}$$

In GA, the individuals are evaluated by fitness function. The fitness value of each individual determines whether it can be passed into the next generation or not. In evolutionary algorithms, penalty function is often used to handle constraints. The fitness function used in this paper is as formula (7). In formula(7), $f(X)$ is the objective function. $\frac{Stress_{max}}{Stress_{lim}}$ and $\frac{Disp_{max}}{Disp_{lim}}$ represent the constraints violation distance. If the individual is feasible these two items are less than "1". Otherwise, they are bigger than "1". The last item, $\frac{perimeter}{4\times f(X)}$, represents the geometric topology influence. Where, $perimeter$ is the length of the geometric topology outline. In this paper, if there is a shared edge or a shared vertex for two meshes, it is defined they are connected. To reduce the influence of this item, it is divided by $4 \times f(X)$. therefore, it is always less than "1".

$$fitness = f(X) + \frac{Stress_{max}}{Stress_{lim}} + \frac{Disp_{max}}{Disp_{lim}} + \frac{perimeter}{4 \times f(X)} \tag{7}$$

Moreover, for constrained problems, there are often some infeasible individuals during evolution. In this study, it is defined that any feasible individual is preferred to any infeasible individual. For infeasible individuals, those with smaller constraint violation are preferred. Therefore, for infeasible individuals, the first item - $f(X)$ is replace with a bigger constant so that the fitness value is bigger than that of any feasible individual.

5. Experiments and Discussions

A number of experiments are performed to demonstrate the effectiveness of the stress-based crossover operator. For all experiments, the design domain is divided by hexahedral mesh. The parameters of GA used in this paper are listed in Table 1. For each problem, the chromosome length is equal to mesh number.

Table 1 GA Parameters

Population Size	Elites	Crossover Rate	Mutation Rate	Tournament Size	Max.Generation
100	1	1	0.01	2	500

In this paper, the following material properties are assumed: Young's modulus $E = 206GPa$, Poisson's ratio $\nu = 0.3$, and density $\rho = 1000kgm^3$.

5.1. Cantilever Problem

The first example is the so-called cantilever benchmark problem as shown in Fig. 3, which has been studied extensively in structure topology optimization. The cantilever dimensions are 20L ×10L, were L = 1 mm. The thickness is 1 mm. The beam is simply fixed at its left and a downward concentrated load $F = 1.0\times10^{10}$ N is applied at the mid-span on the right frame (point A). For this example, the design domain is divided into 20 × 20 = 400 meshes.

For this problem, a comparison of 2-point crossover (2X) and stress-based crossover (SX) was carried out to show the effectiveness of SX in checkerboard pattern suppression. A comparison of ESO and SX was performed to verify the local optimal design problem of ESO and to show the flexibility of SX to constrained problems.

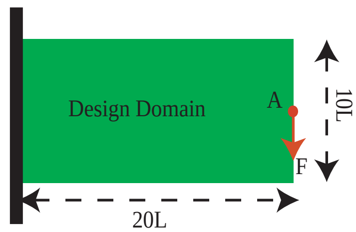


Fig. 3 2D Cantilever Problem

5.1.1. Comparison of 2X and SX For the same constraints $Stress_{lim} = 5.5 \times 10^{10}$ N and $Disp_{lim} = 10$ mm, the geometric results of 2X and SX are shown in Fig. 4 and Fig. 5, respectively. The numerical properties of Fig. 4 and Fig. 5 are listed in Table 2.

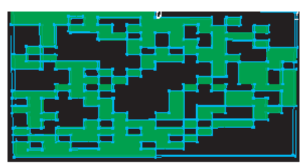


Fig. 4 Solutions of 2X

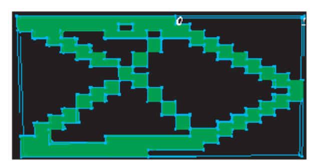
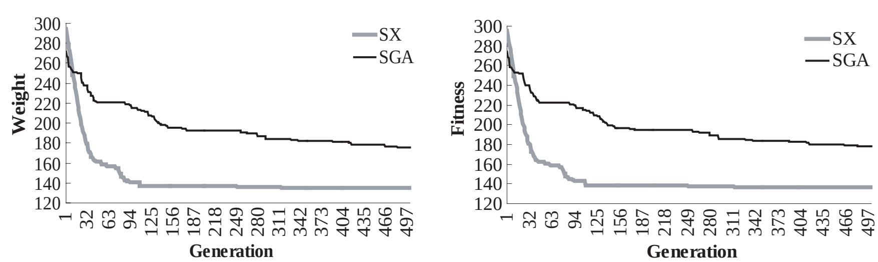


Fig. 5 Solutions of SX

Table 2 Numerical properties of Fig. 4 and Fig. 5

Index	Weight	$Stress_{max}$ (N)	$Disp_{max}$ (mm)
Fig. 4	176(44%)	4.9849×10^{10}	9.9843
Fig. 5	124(31%)	4.3031×10^{10}	9.9788

Both numerical results meet the user-defined constraints. The weight of 2X solution was 176, which was 44% of full material. The weight of SX solution was 124, which was 31% of full material. The numerical results showed that SX can find a much lighter design meeting the constraints. Geometric results comparison indicats that SX can greatly suppress the checkerboard-like material distribution.



(a) Objective function -*weight* evolution history

(b) Fitness evolution history

Fig. 6 Evolution Histories of 2X and SX

Comparisons on objective function *weight* evolution histories and fitness evolution histories in Fig.6 (a) and (b) figure out that SX converges more quickly than 2X. SX converged at about 100 generation and 2X does not converge until final generation. Moreover, the final solution' *weight* by SX is much smaller than that by 2X.

5.1.2. Comparison of ESO and SX ESO, as one common approach to STOPs, is adopted to compare with SX method. The geometric solution with $rrRate = 0.05$ and $erRate = 0.001$ is shown in Fig.7 that is the finally converged solution at generation 14. We set constraints like that $Stress_{lim} = 5.0 \times 10^{10}$ N and $Disp_{lim} = 10.0$ mm for SX to this problem. The final SX geometric solution is shown in Fig. 8. Accordingly, the numerical properties are listed in Table 3.

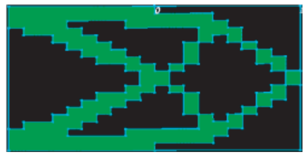


Fig. 7 ESO result of Generation=14

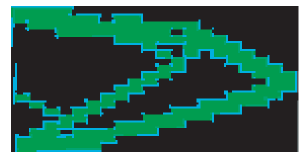


Fig. 8 Solution of SX

At here we will validate the local optimal design problem caused by ESO. For ESO, there is no constraints handling strategies during the evolution. When ESO is applied to solve constrained problems, the evolution will stop once the solution violates the constraints. On the constrary, SX always tries to search out a optimal solution that meets constraints until convergence.

From the geometric solutions of Fig. 7 and Fig. 8 we can know they are difference topologies. The numerical properties in Table 3 demenstrate that the solution of SX with $weight = 33.75\%$, $Stress_{max} = 4.641 \times 10^{10} N$ and $Disp_{max} = 9.803 mm$ is more optimal than ESO solution with $weight = 35\%$, $Stress_{max} = 5.277 \times 10^{10} N$ and $Disp_{max} = 3.196 \times 10^{1} mm$. These just verify Fig. 7 is not the global optimal design.

Table 3 Numerical properties of Fig. 7 and Fig. 8

Index	Weight	$Stress_{max}(N)$	$Disp_{max}$ (mm)
Fig.7	140(35%)	5.277×10^{10}	3.196×10^{1}
Fig.8	135(33.75%)	4.641×10^{10}	9.803

The *weight* convergence history of SX is shown in Fig. 9. From that we can know after 32 generations it converges. Comparing with ESO solution, which is converged in 14 generations , we can know SX converges much slower than ESO. Moreover, ESO uses one individual and SX uses 100 individuals. Therefore, the individuals evaluation times of SX (100 times for every generation in this paper) is much more than that of ESO (one time for one generation).

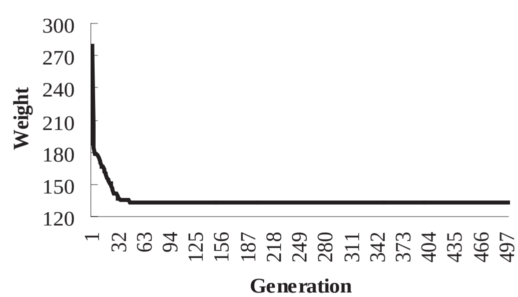


Fig. 9 Weight Evolution History of SX

5.2. Michell-Type Problem

The Michell-type structure is the first truss solution of least weight. A general theory for deriving these structures was published a century ago by the engineer and mathematician A.G.M. Michell(25), and it has been widely used as a typical problem to verify the effectiveness of evolutionary approaches to STOPs. In this paper, it is also adopted to test the capability of

SX with GA for multi-constrained STOPs. The design domain of dimensions 1000 mm × 500 mm shown in Fig. 10 is divided into 20 × 40 = 800 meshes. The thickness of the stucture is 10mm. The two corners at the bottom are fixed and a downward concentrated load F = 1 KN is applied at mid-span on the under frame.

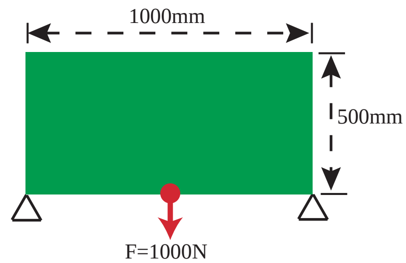


Fig. 10 Michell Problem

For this problem, we perform experiments with two group constraints:

(1) $Stress_{lim} = 0.050$ N, $Disp_{lim} = 1.0 \times 10^{-9}$ mm

(2) $Stress_{lim} = 0.055$ N, $Disp_{lim} = 1.5 \times 10^{-9}$ mm

The geometric results of experiments (1) and (2) are shown in Fig. 11 and Fig. 12. Numerical properties of solutions of Fig. 11 and Fig. 12 are listed in Table 4.

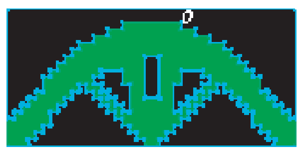


Fig. 11 Solution of Experiment (1)

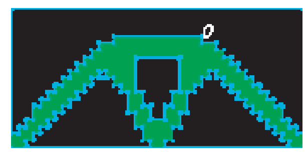


Fig. 12 Solution of Experiment (2)

Table 4 Numerical properties of Fig.11 and Fig.12

Index	Weight	$Stress_{max}$ (N)	$Disp_{max}$(mm)
Fig.11	316(39.5%)	4.99904×10^{-2}	9.95653×10^{-10}
Fig.12	204(25.5%)	5.49431×10^{-2}	1.46536×10^{-9}

The geometric result shown in Fig.11 indicates that GA with SX searches out a topology the same as the theoretical solution with the numerical results meeting user defined constraints. When we set different constraints like group (2), an applicable solution, as shown in Fig.12 is obtianed with numerical values less than the constraints.

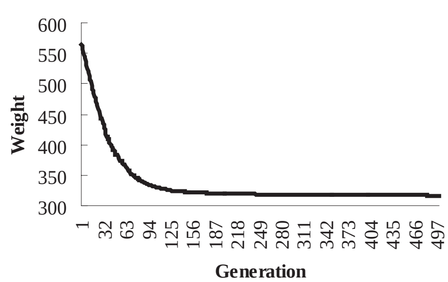


Fig. 13 Evolution History of Experiment (1)

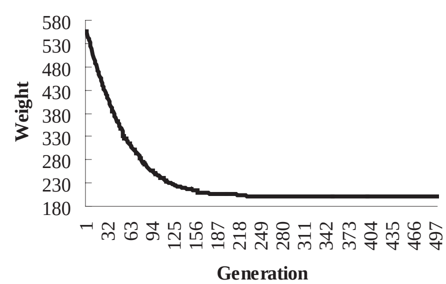


Fig. 14 Evolution History of Experiment (2)

The objective - *weight* evolution histories of experiment (1) and (2) are shown in Fig.13 and Fig.14, respectively. Both demenstrate at the beginning 100 generation the convergence speed is quickly.

5.3. Shear Wall Problem

Fig. 15 shows a multiple load shear problem with a size of 3000 mm × 6000 mm and the thickness is 50 mm used by Yang[26]. For this problem, four loads are applied on the top and middle of two sides and the bottom is fixed. Each load is $F = 5.12 \times 10^9$ N. The design domain is divided into $20 \times 40 = 800$ meshes. For this problem, the top-right load point is displaced instead of displacement of the whole structure.

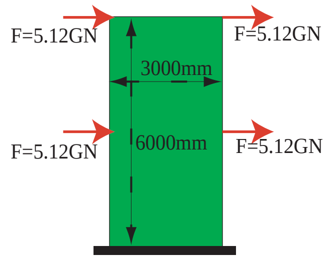


Fig. 15 2D Shear Problem

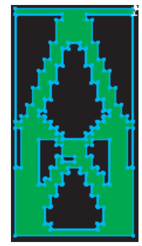


Fig. 16 Solution (1)

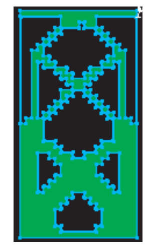


Fig. 17 Solution (2)

Table 5 Numerical properties of Fig.16 and Fig.17

Index	Weight	$Stress_{max}$ (N)	$Diap_{max}$ (mm)
Fig.16	369(46.0%)	3.814×10^{6}	2.3384×10^{-1}
Fig.17	372(46.5%)	3.231×10^{6}	2.4360×10^{-1}

For this problem, the constraints are $Stress_{lim} = 4.0 \times 10^6$ N and $Disp_{lim} = 2.0$ mm. Two trial optimization are done on same condition and same parameters. Two different solutions are shown in Fig. 16 and Fig. 17. Both geometric results are without "checkerboar" pattern and both numerical results are satisfied the constraints. Although the numerical values of $Stress_{max}$ and *displacement* shown in Table 5 are approximately the same, the geometric topologies are different. These observations demonstrate the global search ability of SX.

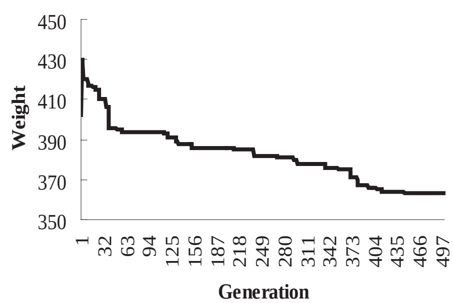


Fig. 18 Weight Evolution History (1)

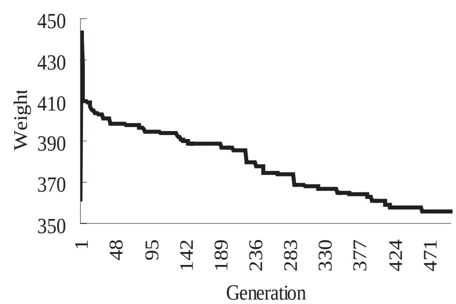


Fig. 19 Weight Evolution History (2)

The *weight* evolution histories of experiment (1) and experiment (2) are shown in Fig.18 and Fig.19. Comparing with proceeding experiment problems, we can know the convergence speed of this multiple loaded problem is different from that of other single loaded problems. For five hundreds generation, it converges almost at 460 generation. Connecting two different geometric solution in Fig.16 and Fig.17 and the *weight* evolution histories in Fig.18 and Fig.19 we can know the solution domain of multiple loaded problem is more complicated than that of single loaded problem.

6. Conclusion

Topology optimization is one of the most challenging fields in structural optimization. ESO is one formal approach for structure topology optimization, which is based on the principle of gradually removing less-stressed elements. However, it is not based on the optimization algorithm principles and is not flexible for multi-constrained problems. GA for structure topology optimization problems has been developed because it is a global search algorithm and is flexible for various complicated problems. However, disconnected geometric topology and checkerboard-like material distribution solutions are always cumbersome problems for simple GA for STOPs. A stress-based crossover operator was proposed to solve these problems. The capabilities of stress-based crossover operators were validated through a number of experiments. A comparison of 2X with SX showed that SX can greatly suppress the checkerboard pattern and obtain a reasonable solution easily. Comparison of SX and ESO indicated that SX is effective for multi-constrained problems. The Michell-type problem experiments verified the effectiveness of GA with SX. Multi-loaded problem experiments demonstrated the global search properties of GA with SX. All experiments demonstrated that SX is powerful for continual structural topology optimization problems.

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