

# Analysis of the Performance of Genetic Multi-Step Search in Interpolation and Extrapolation Domain

Yoshiko Hanada  
Faculty of Engineering,  
Kansai University,  
Osaka, Japan  
hanada@ipcku.kansai-u.ac.jp

Tomoyuki Hiroyasu  
Faculty of Life and Medical  
Sciences, Doshisha University,  
Kyoto, Japan  
tomo@is.doshisha.ac.jp

Mitsunori Miki  
Faculty of Science and  
Engineering, Doshisha  
University, Kyoto, Japan  
mmiki@mail.doshisha.ac.jp

## ABSTRACT

In this paper, we examine overall performances and behaviors of deterministic multi-step search in interpolation / extrapolation domain, dMSXF and dMSMF, using NK model that is one of appropriate models for analyzing fundamental search mechanisms in combinatorial problems. We focus on the local property of landscape, such as epistasis that is comprehended as ruggedness in fitness function, and investigate the efficacy of dMSXF and dMSMF and the behavior observed by tuning the level of epistasis.

## Categories and Subject Descriptors

G.2.1 [Discrete Mathematics]: Combinatorics - Permutations and combinations

## General Terms

Algorithms

## Keywords

Genetic Algorithm, Local Search, Combinatorial Optimization, NK Model

## 1. INTRODUCTION

In combinatorial problem Genetic Algorithms (GAs) actualize an effectual search using genetic operators for inheritance and acquisition of characteristics. These two classes of search, focusing on inheritance or acquisition, are called, respectively, the *interpolation search* and the *extrapolation search* by introducing a distance measure between solutions [1]. Deterministic Multi-step Crossover Fusion (dMSXF) [2] is one of promising interpolation-directed crossover methods based on neighborhood search. Our method, deterministic Multi-step Mutation Fusion (dMSMF), is a complementary search of dMSXF for exploring the extrapolation domain. In our previous study, effectiveness of incorporation of dMSMF into dMSXF was qualitatively demonstrated in Traveling Salesman problem and Job-shop Scheduling Problem [3].

In this paper, we adopt NK model [4] and analyse overall effect of dMSXF and dMSMF in combinatorial problems by tuning epistasis intensity.

## 2. GENETIC MULTI-STEP SEARCH

In this section, the procedures of dMSXF and dMSMF are introduced. dMSXF[2] implements multi-step neighborhood

searches from a parent  $p_1$  in the direction approaching the other parent  $p_2$ , on the other hand, our method, dMSMF, advances the search in the direction that separates from the parents' neighborhood. These methods can be constructed by introducing a problem-specific neighborhood structure and a distance measure. The procedures are described as follows. Here,  $d(p_1, p_2)$  denotes the distance between solutions  $p_1$  and  $p_2$ , and the set of offspring generated by parents  $p_1, p_2$  is indicated by  $C(p_1, p_2)$ .

### Procedure of dMSXF:

0. Let  $p_1, p_2$  be parents and set their offspring  $C(p_1, p_2) = \phi$ .
1.  $k=1$ . Set the initial search point  $x_1 = p_1$  and add  $x_1$  into  $C(p_1, p_2)$ .
2. /Step  $k$ / Prepare  $N(x_k)$  composed of  $\mu$  neighbors generated from the current solution  $x_k$ .  $\forall y_i \in N(x_k)$  must satisfy  $d(y_i, p_2) < d(x_k, p_2)$ .
3. Select the best solution  $y$  from  $N(x_k)$ . Let the next search point  $x_{k+1}$  be  $y$  and add  $x_{k+1}$  into  $C(p_1, p_2)$ .
4. Set  $k = k + 1$  and go to 2. until  $k = k_{max}$  or  $x_k$  equals  $p_2$ .

### Procedure of dMSMF:

0. Let  $p_1, p_2$  be parents and set their offspring  $C(p_1, p_2) = \phi$ .
1.  $l=1$ . Set the initial search point  $x_1 = p_1$ .
2. /Step  $l$ / Prepare  $N(x_l)$  composed of  $\lambda$  neighbors generated from the current solution  $x_l$ .  $\forall y_i \in N(x_l)$  must satisfy both  $d(y_i, p_1) > d(x_l, p_1)$  and  $d(y_i, p_2) > d(x_l, p_2)$ .
3. Select the best solution  $y$  from  $N(x_l)$ . Let the next search point  $x_{l+1}$  be  $y$  and add  $x_{l+1}$  into  $C(p_1, p_2)$ .
4. Set  $l = l + 1$  and go to 2. until  $l = l_{max}$ .

dMSMF would be applied when  $d(p_1, p_2) < d_{min}$  is satisfied, i.e., parents' characteristics are extremely similar to each other, instead of applying dMSXF.

## 3. NUMERICAL EXPERIMENTS

In this study, we analyze in search behaviors of dMSXF and dMSMF using NK model. NK model is a simple and flexible fitness function model of which ruggedness of landscape can be tuned by changing one parameter. The detail description of this model would be found in [4]. This model is represented by a binary string of length  $N$ . The parameter  $K$ , set from 1 to  $(N-1)$ , indicates the level of epistasis, which has intensified impact on the ruggedness of landscape. The bigger  $K$  makes a fitness correlation between neighborhood solutions smaller.

In experiments, NK model of  $N=96$  tuning  $K$  in the range of 2 to 24 are adopted. For overall experiments, the population size  $N_P$  was set to 40, a search was terminated after

50 generations.  $k_{max}$  and  $l_{max}$  was set from 2 to 32. Here, we set  $k_{max}=l_{max}$  and  $\mu = \lambda$ . To enlarge the parameters  $k_{max}$  and  $l_{max}$  makes their diameter of neighborhood at each transition smaller. The number of offspring  $N_C$  generated by each pair of parents was set to 144. The generation-alternation model is same as in the previous works [3].

### 3.1 Behavior against the level of Epistasis

Here, effectiveness on reproduction mechanisms of offspring of both dMSXF and dMSMF is shown to be kept through increases in  $K$  by enlarging  $k_{max}$  and  $l_{max}$ .

Fig. 1 and Fig. 2 demonstrates typical distributions of offspring generated by two methods. Smaller evaluation values are better. The horizontal axis indicates the hamming distance from the parent  $p_1$ . For dMSXF, to highlight the effectiveness of the multi-step search in the interpolation domain, we compared it with Uniform Crossover (UX) that generates a uniform distribution of offspring between parents. Fig. 2 shows offspring generated by dMSMF when the distance between  $p_1$  and  $p_2$  is smaller than  $N * 0.2$ .

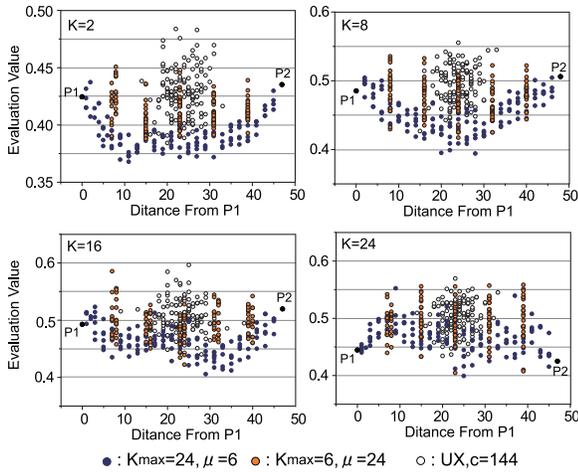


Figure 1: Distribution of offspring of dMSXF

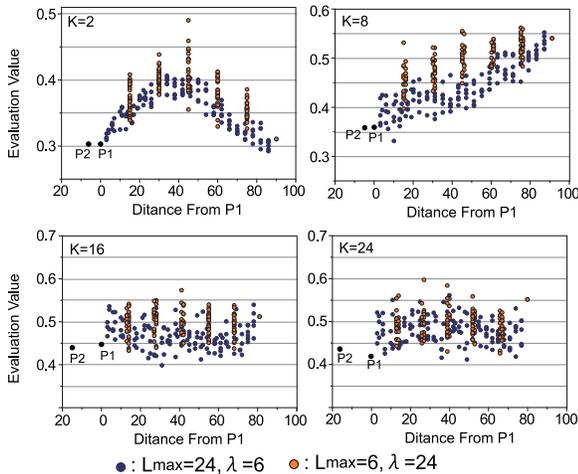


Figure 2: Distribution of offspring of dMSMF

From distributions shown in these figures, it is confirmed that a small  $K$  has a smooth landscape and landscape becomes more rugged in accordance with increase of  $K$ . In Fig. 1, UX generates mostly middest offspring in any rugged

function landscapes, on the other hand, it is anticipated that dMSXF has high potential to generate favorable offspring along the landscape even under high epistasis. From the distribution of dMSMF, by setting  $l_{max}$  bigger, improvement in search performance can be expected in the same mechanism of dMSXF.

### 3.2 Effectiveness of Extrapolation Search

Fig. 3 compares the performance of dMSXF+dMSMF with that of dMSXF. These results show the average of fitness from 200 trials.

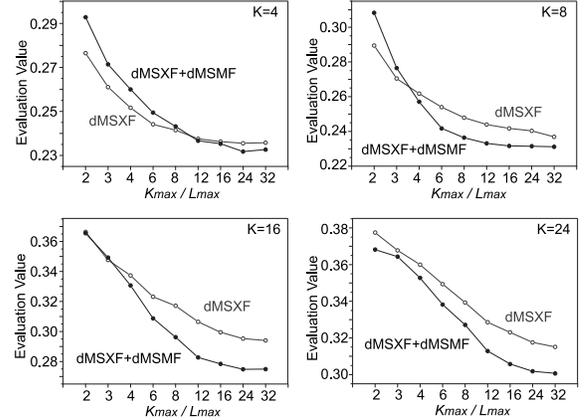


Figure 3: Effectiveness of Extrapolation Search

From Fig. 3 we can see both dMSXF and dMSXF+dMSMF improve the search performance in all instances by enlarging  $k_{max}$  or  $l_{max}$  against increase in  $K$ . When  $K$  is small, i.e., the landscape is smooth, effectiveness of incorporation of dMSMF cannot be observed. However, the landscape becomes complex as increase in  $K$ , dMSXF+dMSMF considerably improves search performances, which indicates rugged landscape strongly requires extrapolation searches.

## 4. CONCLUSIONS

Through experiments focusing on the local property of landscape, effectiveness on reproduction mechanisms of offspring of dMSXF and dMSMF was shown to be kept through increases in the level of epistasis intensity by enlarging the number of steps in multi-step search.

## Acknowledgment

This work was partially supported by Grant-in-Aid for Scientific Research (C), 20560380.

## 5. REFERENCES

- [1] Extrapolation-Directed Crossover for Job-shop Scheduling Problems: Complementary Combination with JOX, Proc. of GECCO 2000, pp. 973–980 (2000).
- [2] Deterministic Multi-step Crossover Fusion: A Handy Crossover for GAs, Proc. of Parallel Problem Solving from Nature, PPSN VII, pp. 162–171 (2002).
- [3] Genetic Multi-step Search in Interpolation and Extrapolation domain, Proc. of GECCO 2007, pp. 1242–1249 (2007).
- [4] : Adaptation on rugged fitness landscapes, Lectures in the Sciences of Complexity, Vol. 1, Addison Wesley (1989).